Numerical approximations of harmonic 1-forms on real loci of Calabi-Yau manifolds

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Abstract: For applications, it is desirable to have Calabi-Yau manifolds of complex dimension three which contain a real three-dimensional submanifold with a nowhere vanishing harmonic 1-form. The harmonic equation depends on the Calabi-Yau metric, which is not known explicitly. Currently, one conjectural example of such a manifold based on the SYZ conjecture exists. In the talk, I will explain a second conjectural example motivated by numerical approximations using neural networks. The numerical approximation works by first computing an approximate Calabi-Yau metric, and then computing a harmonic form for this approximate metric. This is based on an approach by Donaldson for computing approximate Calabi-Yau metrics that I will briefly review. I will also compare the results with a proven non-example, i.e. one where a harmonic 1-form exists, but it is guaranteed to have zeros. Such examples are interesting for M-theory in Physics. This is based on arXiv:2405.19402, which is joint work with Mike Douglas, Yidi Qi, and Rodrigo Barbosa. Time permitting I will comment on using an approximate harmonic 1-form in a numerically verified proof to rigorously show existence of such a 1-form. This is work in progress with Mike Douglas, Javier Gómez-Serrano, Yidi Qi, and Freid Tong. ▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

Experimental results: 1-forms and their zeros

- 1. Fermat: non-example 1; no harmonic 1-form.
- 2. Random Quintic: non-example 2; harmonic 1-form must have zeros
- 3. **CICY1:** large perturbation $\epsilon = \frac{1}{4}$, harmonic 1-form may have zeros
- 4. **CICY2:** small perturbation $\epsilon = \frac{1}{100}$, conjecture no zeros



Monge-Ampere loss near singularities



From left to right: CICY1, CICY2, then two plots for Quintic y-axis: the Monge-Ampère error $|1 - \omega^m / \Omega \wedge \overline{\Omega}|$ x-axis: "approximate distance to a singular locus", more precisely:

▶ for CICY1 and CICY2 as
$$\min\{d_2(x)/|x|^2, d_4(x)/|x|^4\}$$
, where $d_2(x) = \max\{|x_4^2 + x_5^2|, |x_0|^2, |x_1|^2, |x_2|^2, |x_3|^2\}$ and $d_4(x) = \max\{|x_1^4 + x_2^4 + x_3^4|, |x_0|^4, |x_4|^4, |x_5|^4\}$

• Quintic is smoothing of two singularities: (a) intersection of divisors
$$Z(f)$$
 and $Z(g^*)$, (b) an ordinary double point at $a = [1:0:0:0:0]$ third image: approximation of the distance to (a), namely $\max\{|x_0^2 + \cdots + x_4^2|/|x|^2, f^*(x)/|x|^3\}$ fourth image: approximation of the distance to (b), namely $|x - (|x|, 0, 0, 0, 0)|/|$

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• Quintic is smoothing of two singularities: (a) intersection of divisors Z(f) and $Z(g^*)$, (b) an ordinary double point at a = [1:0:0:0:0] third image: approximation of the distance to (a), namely $\max\{|x_0^2 + \cdots + x_4^2|/|x|^2, f^*(x)/|x|^3\}$ fourth image: approximation of the distance to (b), namely |x - (|x|, 0, 0, 0, 0)|/|.

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Formation of long necks



x-axis: approximation of the distance to the singular locus of the initial singular variety

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Formation of long necks



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Plots of approximately harmonic 1-forms



CICY2 (close to singular limit): λ almost zero in S²-direction, parallel in S¹-direction

1-form on CICY1 with z1 = 0, z2 = 0 (Scaled by 100)



1-form on CICY1 with z1 = 0, z4 = 0 (Scaled by 0.5)

